J. Fluid Mech. (1996), vol. 321, pp. 315–333

Experiments on double-diffusive sugar–salt fingers at high stability ratio

By JOHN R. TAYLOR[1] AND GEORGE VERONIS[2]
[1]School of Physics, University College, UNSW, ADFA, Canberra ACT 2600, Australia
[2]Department of Geology and Geophysics, Yale University, New Haven, CT 06511, USA

(Received 20 September 1995 and in revised form 31 March 1996)

In a series of laboratory experiments the growth of double-diffusive salt fingers from an initial configuration of two homogeneous reservoirs with salt in the lower and sugar in the upper layer was investigated. For most of the experiments the stability ratio was between 2.5 and 3, where the latter value is at the upper limit (the ratio of salt to sugar diffusivities) for which fingers can exist. In these experiments long slender fingers are generated at the interface. Essentially all theories or physical bases for models of salt fingers presuppose such a configuration of long fingers. Our measurements show that the length of fingers at high stability ratio increases with time like $t^{1/2}$, with a coefficient that is consistent with the diffusive spread of the faster diffusing component (salt). When the initial stability ratio is closer to unity, fingers penetrate into the reservoirs very rapidly carrying with them large anomalies of salt and sugar which give rise to convective overturning of the reservoirs. The convection sweeps away the ends of the fingers, and when it is intense enough (as it is when the sugar anomaly is large) it can reduce the finger height to a value less than the width. After this initial phase the finger length grows linearly with time as has been found in previous studies. These results show that salt fingers can evolve in quite different ways depending on the initial stability ratio and must cast doubt on the use of simple similarity arguments to parameterize the heat and salt fluxes produced by fingers.

1. Introduction

Salt fingers form when a layer of fluid is stably stratified by heat and unstably stratified by salt and when the vertical density distribution is gravitationally stable. Typically in laboratory experiments the fingers develop from an initial configuration with two homogeneous layers, the upper one warm and salty and the lower one cool and fresh. Less frequently the initial vertical distributions are continuous. In the Atlantic Ocean east of Barbados (Schmitt 1987) there is a horizontally extensive region with overall stratification appropriate for the formation of salt fingers but with several homogeneous layers stacked vertically and separated by salt finger ‘interfaces’. In both laboratory and ocean, although the vertical density profile is gravitationally stable, the distribution of salt is unstable. The potential energy of the salt distribution is released when a blob of salty fluid is displaced downward and loses its heat through diffusion. The salt diffuses much more slowly so the blob retains its salt, becomes heavier than its surroundings and continues its downward motion. There is a corresponding upward flow when a fresh particle is displaced upward.

The original stability analysis (Stern 1960) was for the heat–salt system but the same reasoning applies to a system with two solutes: an unstably stratified slower diffusing solute and a stably stratified solute with a larger diffusion coefficient. Since heat losses to the environment are difficult to control, laboratory experiments are often carried out with two solutes, usually salt and sugar instead of heat and salt. Salt diffusivity is only three times that of sugar whereas the ratio of heat to salt diffusivities is about 80. Nonetheless, many of the qualitative features of the heat–salt system are exhibited by the salt–sugar system.

The configuration most often invoked by the words 'salt fingers', is one in which the fingers are vertically very long (compared to their width) and quasi-steady (figure 1). The finger zone may or may not be sandwiched between convective layers. In an actual experiment the salt fingers may be not so long and/or the concentrations in the reservoirs above and below may be continuously stratified or homogeneous, or the fingers may be not much taller than their width (cf. Griffiths & Ruddick 1980, figure 5). In this case, there are almost invariably convecting layers above and below.

In spite of these easily observable differences there has been a tendency to treat salt fingers as a single phenomenon, potentially amenable to a single analysis. Normally the only independent parameter used to report laboratory results is the stability ratio, $R_\rho = \alpha\Delta T/\beta\Delta S$, where $\rho_0\alpha\Delta T$ is the stabilizing increment of density due to temperature, T, and $\rho_0\beta\Delta S$ is the destabilizing increment due to salt, S. Here α and β are the coefficients of thermal expansion and salinity contraction respectively and ρ_0 is a reference density. (Sometimes vertical gradients instead of increments are appropriate.) Yet it is evident that ΔT and ΔS may be varied independently.† Theoretical models almost always assume that the fingers are long, thin, quasi-steady features. However, analyses based on that assumption are applied to experiments in which the fingers are no taller than they are wide.

The original theoretical model of a salt finger was given by Stern (1960, 1975). He assumed uniform vertical gradients, $\mathrm{d}\overline{T}/\mathrm{d}z$ and $\mathrm{d}\overline{S}/\mathrm{d}z$, for the basic state and showed that there is a steady two-dimensional solution of perturbed T, S and vertical velocity, w, with relatively warm, salty, descending fingers interspersed between cool, fresh, ascending fingers. (The following model assumes a heat–salt system, even though sugar and salt were used in our experiments.) For equilibrium (steady) fingers the heat balance is

$$w\frac{\partial\overline{T}}{\partial z} = \kappa_T \nabla_h^2 T$$

and the vertical momentum balance is

$$\rho(\beta S - \alpha T) = \nu\nabla_h^2 w$$

where $\partial\overline{T}/\partial z$ is the horizontally averaged vertical temperature gradient, ∇_h^2 is the horizontal Laplacian, ν is kinematic viscosity, and κ_T is the molecular diffusivity of temperature. Howard & Veronis (1987) obtained results qualitatively similar to Stern's for a fluid with negligible κ_S, where κ_S is the molecular diffusivity of salt, when a finger interface is sandwiched between uniform reservoirs above and below.

In both analyses the descending salty fluid and ascending fresh fluid provide the driving force in (2). The stabilizing effect of the mean temperature gradient is short-circuited when heat is transferred from descending to ascending fingers by horizontal diffusion in (1). Even when the vertical scale of the fingers is not larger than

---

† The magnitude of ΔT or ΔS is embedded in discussions involving the 4/3 law and not on the effect of variations of ΔT and ΔS on the evolution of the system.

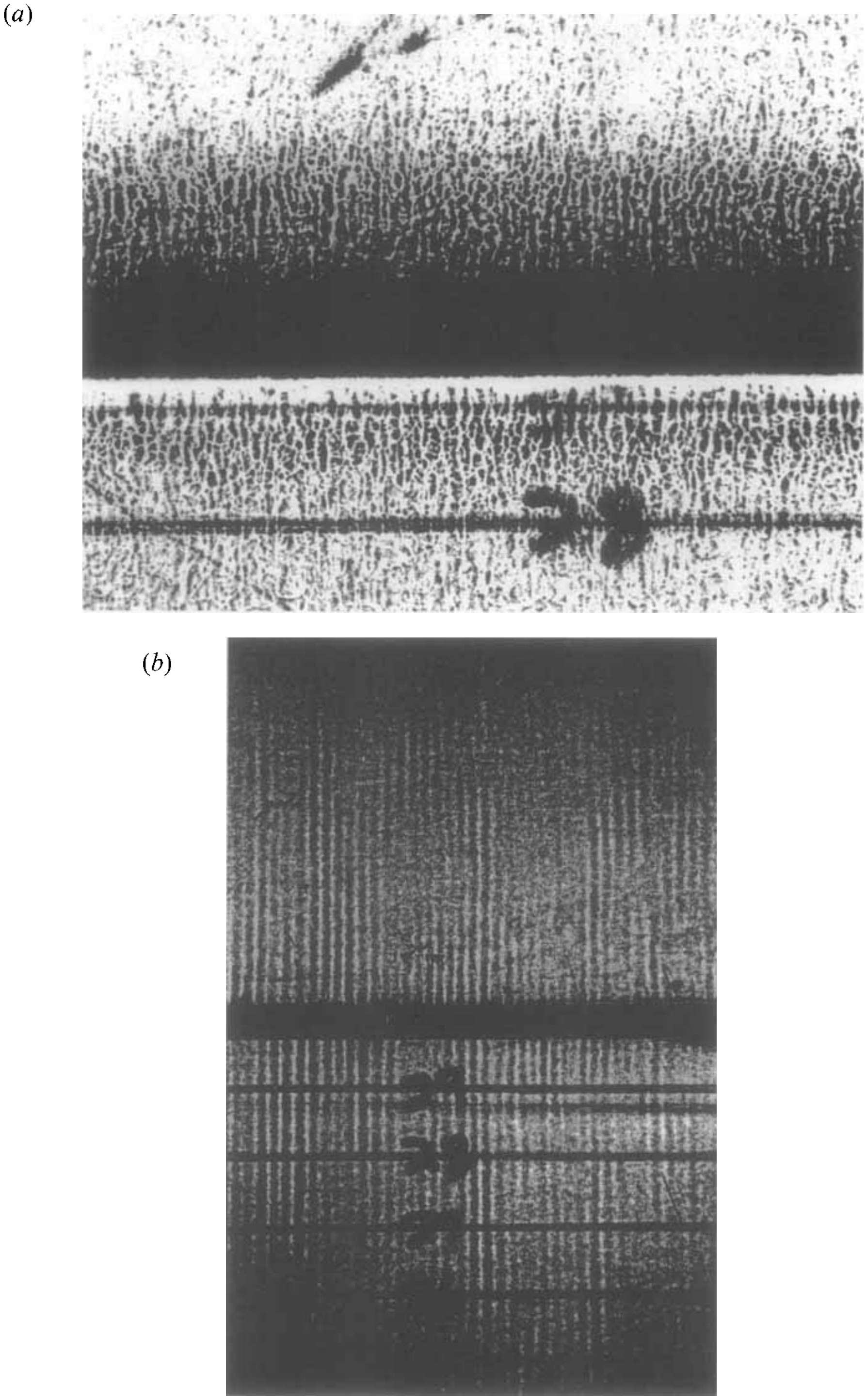

FIGURE 1. Two shadowgraph images of salt fingers taken in experiments with initially high R_ρ. Image (*a*) was taken 43 minutes after the removal of the barrier in an experiment with the same initial conditions as experiment 9 (table 1) and shows the tiled or beaded structure of the fingers, while (*b*) shows long straight fingers in the later stages of experiment 11. The horizontal lines are 1 cm apart and the dark thick horizontal line is the shadow of the barrier.

the horizontal scale, and even when the system is strongly nonlinear, the balances expressed in (1) and (2) must be at least partially satisfied because the stabilizing effect of the temperature must be short-circuited to release the potential energy of the salt.

Stern (1976) used upper-bound theory, together with a set of plausible physical assumptions for the heat–salt case and the results of his (Stern 1969) collective instability analysis, to obtain a flux ratio, R_f, of 1/4, where $R_f = \alpha F_T / \beta F_S$, and F_T and F_S are the vertical, convective fluxes of temperature and salt respectively (the equivalent value for the salt-sugar system is 0.5). Howard & Veronis (1987) obtained the same value for a fluid with negligible salt diffusivity by imposing the constraint that the buoyancy flux be maximized. In all salt finger systems the flux ratio is less than unity, i.e. there is a downward flux of density. Laboratory experiments yield flux ratios that are invariably larger than those that are found in these theoretical analyses. Turner (1967) reported a value of $R_f = 0.56$ for $R_\rho > 2$ for heat–salt fingers. The values from experiments by Schmitt (1979), McDougall & Taylor (1984) and Taylor & Bucens (1988) range between 0.4 and 0.7. Experiments on the salt-sugar system by Lambert & Demenkow (1972) and Griffiths & Ruddick (1980) yield flux ratios from 0.88 to 0.94.

It is important to note that the higher values of flux ratios obtained experimentally occur because the theories are based on assumed optimizations (maximum growth rate, maximum buoyancy flux) that may not be realized in the experiments. Furthermore, experimental set-ups vary. At one extreme, if the heat–salt system is gravitationally very stable with a salt concentration just large enough to initiate fingers, the finger array could achieve a certain height and then be dissipated by lateral diffusion without redistributing the salt beyond the limited height of the array. In this case the convective fluxes into the reservoirs will be zero while the overall flux ratio will be determined by vertical diffusion and will equal $\tau^{-1} R_\rho > 1$ (where $\tau = \kappa_S / \kappa_T$ is the ratio of the molecular diffusivities of the two components) so that it would not even be considered double diffusive. Salt–sugar fingers which evolve in this way were found in the present experiments. At this extreme ($R_\rho \approx \tau^{-1}$), but for slightly smaller values of R_ρ, very long quasi-steady fingers form so that the theories of Stern (1975) or Howard & Veronis (1987), both of which give a flux ratio of 1/4, apply. At the other extreme ($R_\rho \approx 1$), a large destabilizing concentration of salt in a marginally stable system will generate relatively intense fingers that lead to strongly convecting layers above and below the interface. The convection will chop off the ends of the fingers, reducing the vertical scale to a value perhaps even smaller than the finger width. In this case, the flux ratio could approach unity since very little of the stabilizing heat need diffuse laterally before the destabilizing salt causes fingers to form, so that the fingers transport a substantial amount of heat in addition to the salt. Between these two extremes, fingers of finite length bounded by convecting reservoirs above and below could lead to a flux ratio between 1/4 and unity, depending on the values of the parameters. In short, for two specified components, e.g. heat and salt, there is no single 'correct' value for the flux ratio just as there is no single 'correct' length for the fingers.

In the set of experiments described below we have attempted to explore the behaviour of salt-sugar systems that are initially two–layer and for which the initial gravitational stability due to the concentrations in the reservoirs range from very strong ($R_\rho \approx 3$) to moderate ($R_\rho \approx 1.5$) and in which the destabilizing sugar in the top layer ranges from large to small concentration.

The results obtained from the experiments are reported after a description of the laboratory set-up. In §3.1 we discuss the evolution of finger length with time for the different stability ratios and destabilizing concentrations. The information about the finger lengths comes from direct visual measurements, from vertical profiles of conductivity, from time-lapse films of the shadowgraph image and from the streaks left by crystals of potassium permanganate dropped from the top. A qualitative description of the behaviour of the system focuses on the evolution of finger length for different combinations of salt and sugar concentrations. Generally speaking, we find that a very stable ($R_\rho \geqslant 2.5$) system gives rise to long fingers and a moderately stable ($1.5 \leqslant R_\rho \leqslant 2.0$) system initially generates short fingers. Most of the experimental studies of salt–sugar fingers reported in the literature are for moderately or weakly stable systems. Most of our experiments deal with very stable systems so that the results complement the earlier ones. Time rates of change of the concentrations of salt and sugar in the reservoirs yield estimates of the vertical fluxes through the finger zone and the flux results are discussed in §3.2.

Our findings are consistent with known physical balances of salt fingers but they depart somewhat from those of earlier studies. The main reason for the departure is that we have identified how low and high values of R_ρ and small and large sugar concentrations lead to differences in the way that the finger length changes during an experiment and in the characteristics of the gradient of salt within the fingers. There are also indications of differences in the fluxes between high- and low-R_ρ experiments although definitive conclusions are restricted by the large uncertainties in the measured fluxes. However, an overall picture arises that, although the basic physics of the fingers is well-understood, some confusion has arisen because of the tendency to treat all finger systems in the same way. By describing the different behaviours we hope that we have supplied evidence that will help to develop more accurate theories and interpretations.

In addition, with the initial value of R_ρ held fixed the value of $\beta\Delta S$ was varied by an order of magnitude to make it possible to determine the evolution of finger growth as a function of the second of the two forcing parameters. As it turns out, the effect of R_ρ is the more important. However, when R_ρ is large, a large value of $\beta\Delta S$ tends to shift the behaviour toward that associated with low stability ratio so the effect of $\beta\Delta S$ is not negligible.

2. The laboratory experiment

The experiments were performed in an acrylic tank with inside base dimensions of 15 by 30 cm. At a height of 30 cm above the base a metal barrier could be slid horizontally into place to partition the tank into two layers. The barrier and its seals followed the design described by McDougall (1981). The overall height of the tank was 70 cm, thus permitting the tank to be filled with two layers of equal depth. The experimental procedure was first to fill the lower layer with a salt solution, slide the barrier into position and then (after removing any salt solution above the barrier) fill with a sugar solution. Once the tank was filled, a polystyrene lid with a small cutout for probes was placed on top of the tank to minimize evaporation from the free surface. The salt fingers were then initiated by carefully withdrawing the barrier from between the two reservoir layers. As has been described, the evolution of the fingers was then followed visually by simple shadowgraphs, and by observing the distortion of dye streaks produced by dropping potassium permanganate crystals into the tank. The change in reservoir properties was monitored by withdrawing samples for later analysis through a multiport sampling tube placed horizontally within each reservoir. The vertical profiles of conductivity and temperature in the gradient or salt finger zone were also recorded by traversing a PME microconductivity probe (Head 1983) and microbead thermistor through the fingers. These conductivity profiles and the extent to which conductivity represents the salinity profile through the fingers is central to our interpretation of these experiments and will be discussed below.

The density of the samples from the reservoirs was determined with an Anton Paar density meter with a relative uncertainty of $\pm 10^{-2}\,\mathrm{kg\,m^{-3}}$. To determine the salt and sugar concentrations in the reservoirs the measurement of an additional physical property is required (Ruddick & Shirtcliffe 1979). We found that the most accurate determination of these concentrations resulted from using the measured density and a value for the reservoir conductivity found by averaging the section of each conductivity profile within the reservoir. For each run the conductivity probe was calibrated against the density meter by forcing the upper reservoir in the initial profile to have zero conductivity and the lower reservoir to have the conductivity calculated from the initial density sample. Independent determinations of the sugar and salt concentrations using measurements of the optical activity of the solution showed that the lower sample always had a negligible sugar concentration and the upper a negligible salt concentration. The known salinities of the upper and lower reservoirs could then be used to give a linear calibration for the conductivity probe. However, a detailed calibration with a range of salinities covering the full output range of the conductivity probe showed that the relationship between conductivity and output voltage was significantly nonlinear. This nonlinearity could be accounted for by using a parabolic fit to the detailed calibration data but there was still a slow drift of the probe's sensitivity during the period of the experiments which had to be accounted for. To do this, the second-order coefficient in the calibration curve was assumed to be constant and the linear and constant coefficients were adjusted to fit the initial upper- and lower-layer properties. Using this procedure gave a linear coefficient of the parabolic calibration curve that changed monotonically over the period of the experiments.

Based on the resolution of the analog-to-digital converter used in recording the profiles we estimate a relative uncertainty in our specific conductivities, c, of $\pm 8 \times 10^{-3}\,\mathrm{S\,m^{-1}}$. The term relative uncertainty is used here because the absolute accuracy of the density measurements against which the conductivity was calibrated is not known. However, in all results to be discussed changes or differences in properties, rather than the properties themselves, are used so that some error in the absolute density is acceptable. The uncertainties in ρ and c translate into an uncertainty in the concentration of salt (expressed as mass of salt per unit mass of solution), T, of $\pm 6 \times 10^{-5}$. Because sugar is non-ionic the conductivity of a solution does not depend directly on its sugar concentration (Ruddick & Shirtcliffe 1979). However, the presence of sugar affects the mobility of salt ions, so the amount of sugar does reduce the conductivity of a solution of salt and sugar relative to that of a solution of the same concentration of salt only. Thus, the translation of the measurement uncertainties into uncertainties in the sugar concentration S depends on the actual S and T concentrations of the solutions. In our experiment for a typical value of $\partial S/\partial c$ of $1.4 \times 10^{-2}/(\mathrm{S\,m^{-1}})$, the error in S will be $\pm 1 \times 10^{-4}$. The errors in S and T given here are considerably smaller than those given by Ruddick & Shirtcliffe (see their table 1) but this may be accounted for by the greater precision of our conductivity measurements, and also the generally lower concentrations of sugar and salt in our experiments. This second point follows because the uncertainty in conductivity quoted by Ruddick & Shirtcliffe was a fixed fraction of the conductivity of their samples.

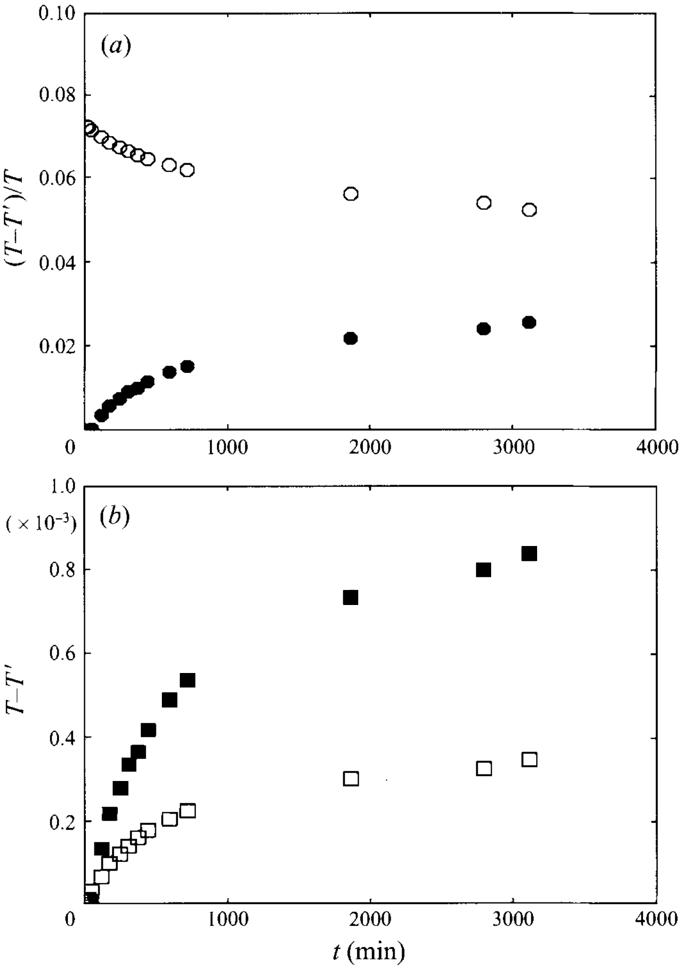

FIGURE 2. Error in the reservoir salinities resulting from the neglect of the effect of sugar on the conductivity of a solution: (*a*) the relative error, (*b*) the absolute error. T is the salinity derived using the method and data of Ruddick & Shirtcliffe (1979), T' is the salinity derived in the same way but neglecting the effect of sugar on conductivity. Filled symbols are points from the lower layer, unfilled from the upper layer. Note that the full inversion calculation was used in all reservoir flux calculations.

As foreshadowed above the final question to be considered in this section is how well our conductivity profiles represent the salinity profile. As described above the conductivity of a salt solution can be significantly reduced by the presence of sugar. However, the magnitude of this reduction depends approximately linearly on the sugar concentration (Ruddick & Shirtcliffe 1979). It happens that, because most of the present experiments started at high R_ρ, the salt concentration in the upper reservoir was always small relative to that in the lower reservoir. This meant that the conductivity in the upper reservoir was small. As a result, the large sugar concentration in the reservoir only caused a significant decrease in what was already a small conductivity. Conversely, in the lower reservoir the sugar concentration was always small (relative to the overall sugar contrast) and the presence of a small amount of sugar reduced the conductivity only slightly compared to what it would have been in a pure salt solution. The overall effect is that the fractional error in a

No.	$\Delta\rho_T(t=0)$	$\Delta\rho_S(t=0)$	$R_\rho(t=0)$	R_ρ (end)
1	56.24	18.8	2.99	2.99
2	38.38	18.4	2.052	2.771
3	46.67	18.61	2.482	2.72
4	23.43	9.097	2.551	2.61
5	12.06	4.605	2.62	-
6	4.757	1.724	2.75	-
7	3.741	1.695	2.21	2.606
8	28.33	18.45	1.54	2.543
9	90.0	36.72	2.45	2.58
10	2.717	1.732	1.57	2.81
11	89.97	30.0	3.0	3.0

TABLE 1. Reservoir density increments in kg m^{-3} and values of R_ρ for the 11 experiments.

salinity profile derived by neglecting the effect of the (unknown) sugar profile is small and does not vary by more than a factor of two over the full depth of the profile. This point is illustrated by figure 2, which shows the difference between the reservoir salinities calculated for experiment 8 with and without including the effect of sugar on the conductivity. The maximum error in the salinity for this experiment caused by neglecting the effect of sugar on conductivity was 8.4×10^{-3}, corresponding to a relative error of 0.026. Errors in salinity of this magnitude were not significant in our interpretation of the salinity gradients within the fingers.

3. Experimental results

3.1. *Finger lengths*

As these transient experiments run down, the stability ratio increases toward τ^{-1}, the maximum value for which fingers occur. In all of our experiments long fingers were established in the very late stages after the concentration anomalies had been decreased by finger activity. The principal differences among the experiments show up in the earlier stages just after the barrier is withdrawn. In very stable configurations (high R_ρ) there is a long (up to an hour) period of diffusive transport in the vertical direction when finger activity is either absent or barely discernible. Less stable configurations lead to immediate formation of salt fingers, which start out as as fine, needle-like filaments and quickly develop into wider, longer fingers. In all cases the barrier was withdrawn slowly with sufficient care to keep the disturbance at the interface small so that any direct effect of the induced motion on the fingers was transient and small.

The data base comes from the eleven experiments listed in table 1. Most of the experiments involved high R_ρ (≈ 2.5) from the outset since that is the range of R_ρ in which long, thin fingers evolve smoothly. Runs 8 and 10 had $R_\rho = 1.5$ initially and run 2 had $R_\rho = 2$. These runs provide a contrast to the high-R_ρ cases. Most experimental studies for salt–sugar reported in the literature focus on values of R_ρ less than 2.

The simplest configuration for finger evolution occurs for a value of R_ρ near τ^{-1}. In this case, salt is diffused vertically into a smooth, very stable, mean gradient through which fingers try to penetrate. Figure 3 plots finger length (as seen by shadowgraphs, square symbols for $t < 7500$ minutes) *vs.* time in experiment 11 in which $R_\rho = 3$

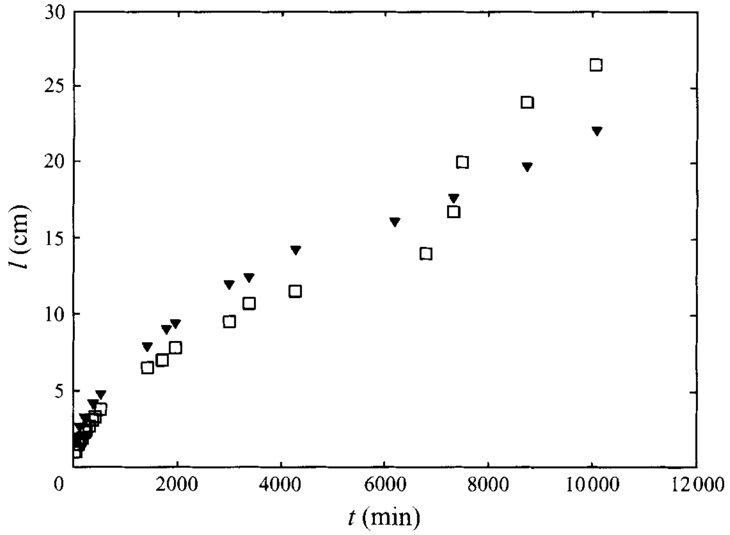

FIGURE 3. Finger length as seen by shadowgraph images (squares) and the thickness the gradient zone as determined from conductivity profiles (triangles) *vs.* time for experiment 11. For $t > 7500$ min the lengths indicated by squares are from breaks in the vertical streaks left by falling potassium permanganate crystals and they exceed the ones obtained from the conductivity profiles.

and $\Delta\rho_S = 30.00\,\mathrm{kg\,m^{-3}}$. Also shown are the gradient zone thicknesses from the conductivity profiles which indicate that visible fingers are consistently shorter than the gradient zone. (The shadowgraph method relies on contrast due to differences in refractive index. The contast becomes too small to be visible near the ends of the fingers.) The fingers appear first as hair-thin, short striations which grow very slowly. A couple of hours after the removal of the barrier the fingers achieved a length of about 1.5 cm. The shadowgraph image showed fingers that are tiled (beaded) with a vertical scale comparable to the width (figure 1). This is consistent with stability analyses (Holyer 1984; Veronis 1987 and Howard & Veronis 1992) which predict that for long, thin fingers the most unstable mode is a disturbance with a vertical scale roughly equal to the finger width. For this very stable stratification the fingers remained vertical with tips that gradually fade into the reservoirs above and below. There was no indication of any horizontal motion near the finger ends until weak convection appeared toward the end of the week-long experiment. This weak overturning circulation appears to have been caused by heat transfer through the side walls and top of the tank due to small temperature fluctuations of air in the laboratory. After 7500 minutes the lengths given by the square symbols on figure 3 were determined from inflections in dye streaks. These lengths are close to or a little greater than the thickness of the gradient zone determined from the conductivity profiles so they do not represent the finger length well. Experiment 1 which had almost the same initial R_ρ evolved essentially identically to experiment 11.

Generally, the effect of molecular diffusion dominated this experiment. It generated a vertical, stable profile of salinity which kept the vertical velocity in the fingers so weak that lateral diffusion had time to wipe out even the sugar anomaly near the ends of the fingers so that little or no sugar anomaly was deposited in the reservoirs and no large-scale convection developed.

Reducing R_ρ to 1.54 (experiment 8) with the same amount of sugar in the upper layer drastically alters the behaviour of the system. The lower stability enables the

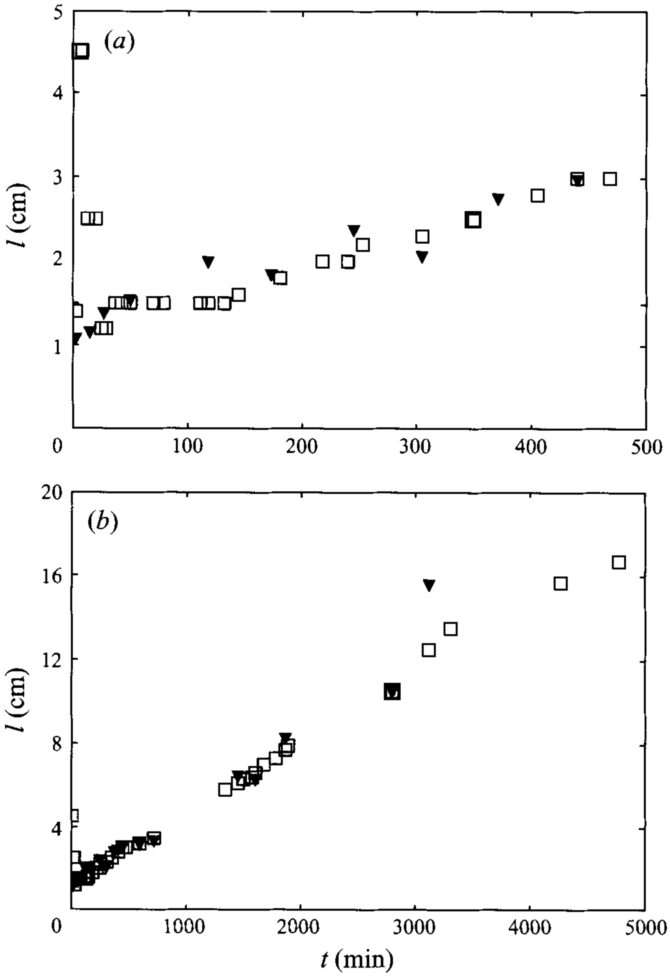

FIGURE 4. (*a*) Finger length *vs.* time near the beginning of experiment 8. Note the rapid penetration of fingers into the reservoirs shortly after $t = 0$ and then the reduction of finger length by convection immediately afterwards. (*b*) Finger length *vs.* time for all of experiment 8. Note the nearly linear dependence of finger length on time during the greater part of the experiment. Symbols as in figure 3.

fingers to penetrate into the reservoirs much more quickly (figure 4*a*). Three minutes after the removal of the barrier the fingers had grown to a length of 1.5 cm and within eight minutes the length was more than 4 cm in some locations and considerably shorter in others. We were observing the penetration of 'blobs' that would form overturning eddies and this finger length is much greater than the thickness of the gradient zone between the two reservoir layers (shown by the triangles on the figure). During this rapid penetration enough sugar anomaly was transported vertically and deposited at the ends of the fingers to generate intense, large-scale convection in the reservoirs which then began sweeping away the ends of the fingers. The length of the fingers was reduced to 2.5 cm 13 minutes after the experiment began and to 1.2 cm by 25 minutes. Then for two hours or so convection kept the finger length at about 1.5 cm. By that time quite a lot of sugar and salt had been transported across the interface and the anomaly that drove the convection was reduced significantly. In addition, the

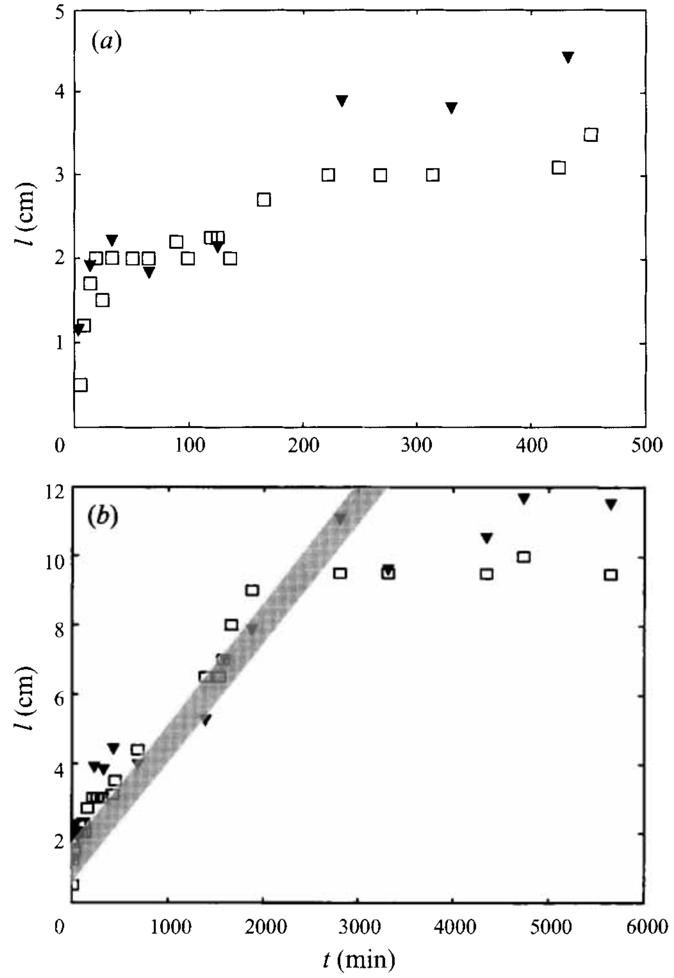

FIGURE 5. (*a*) Finger length *vs.* time near the beginning of experiment 10. The growth of fingers with time does not show the pronounced maximum found in experiment 8 (figure 4*a*) because of the smaller value of ΔS. (*b*) Finger length *vs.* time for all of experiment 10. The length grows linearly with time during the first day or so and then levels off because convection chops off the finger ends. The shaded area shows the finger length versus time for experiment 8 (figure 4*b*) which had a similar initial R_ρ but larger $\beta\Delta S$.

density difference across the finger zone had become greater and it was more difficult for the convective motion in the reservoirs to penetrate into the stably stratified finger zone. As a result, the fingers slowly began to lengthen again with the rate of increase reflecting a partial balance between the penetration of the fingers into the reservoirs and the sweeping away of the finger ends by convection. From this point on the length grew approximately linearly with time (figure 4*b*) until the experiment was terminated (after a little more than 3 days). During this evolution the length of the fingers agreed with the thickness of the gradient zone (derived from conductivity profiles). The linear time dependence after an initially rapid penetration is characteristic of systems with values of the stability ratio less than 2 (see Stern & Turner 1969; Linden 1973).

With the amount of destabilizing sugar reduced by a factor of 10 ($\Delta\rho_S = 1.7\,\mathrm{kg\,m^{-3}}$) for a system with small initial R_ρ (= 1.57, experiment 10) the finger length quickly grew to 2 cm and stayed at that value for a couple of hours before growing to a relatively stable value of 3 cm where it remained for another 3 or so hours (figure 5*a*). Then there was linear (with time) growth for a day (figure 5*b*) as the length advanced to about 9.5 cm. For the next three days convection in the reservoirs was strong enough to keep the fingers from penetrating any farther and the length levelled off to values between 9.5 and 12 cm (figure 5*b*). This limiting of the finger growth is unique to this experiment. It is possible that the small amount of destabilizing sugar in this experiment, especially after much of the sugar had already been transported to the lower reservoir, generated such weak vertical motions in the fingers that they could not penetrate into the reservoirs during the later stages of the run.

In both experiments 8 and 10 the thickness of the finger region grew linearly with time. Within the finger zone the horizontally averaged, vertical salt gradient (as determined from the conductivity profiles) was also constant with z. In contrast, the experiments that started off with large values of R_ρ (2.5 or higher) had salinity gradients that were better described by a Gaussian. This is a characteristic difference between experiments with low and high R_ρ values.

Thus, at low values of R_ρ relatively large $\Delta\rho_S$ leads to: rapid growth of finger length; convection in the reservoirs that reduces the length; a period of fixed length of fingers in which there is a balance between finger penetration and convective cut-off; and then linear growth of fingers. Those are all features associated with the strong motions that result from the large destabilizing stratification of the sugar. When the initial R_ρ is the same but $\Delta\rho_S$ is much smaller, the transitions are smoother. There is no initial overshoot and the later stages reflect a balance between relatively weak penetration and weak convection. In both cases with low R_ρ there is a period of linear growth of finger length.

Table 1 includes the initial values of $\Delta\rho_S$ for experiments 3, 4, 5 and 6 with R_ρ near 2.5. For these high-R_ρ runs the finger lengths grew smoothly from the outset and in all of them convection sheared the finger tips toward the horizontal. It was more difficult to detect the ends of the fingers when $\Delta\rho_S$ was small but horizontal boundaries between the finger zone and the convecting layers could be detected by a break in the streak left by a crystal of $KMnO_4$ dropped from the top. In the low-R_ρ experiments there was a qualitative difference in behaviour when $\Delta\rho_S$ was changed by a factor of 10. The four experiments with R_ρ near 2.5 differed only quantitatively. The finger heights after 2500 minutes were 14.6, 13.5, 11, and 8 cm in experiments 3, 4, 5 and 6 respectively. (Fluid leaked out of the tank through the seal for the barrier in experiment 6 so the correct finger length must be somewhat larger than 8 cm.) As one would expect, fingers driven by smaller concentrations of $\Delta\rho_S$ penetrate less deeply into the reservoirs.

Figure 6 contains a log-log plot of finger length *vs.* time for experiments 3, 4, 5, and 6 all of which started with $R_\rho \approx 2.5$. Some of the data near $t = 0$ were omitted because of the mixing that accompanied removal of the barrier. The figure contains the line showing the $t^{1/2}$ behaviour which characterizes purely diffusive systems. The time evolution of finger lengths from experiment 11 and experiment 1 is accurately described by a $t^{1/2}$ power law. In both of these experiments $R_\rho(\approx 3)$ was so near τ^{-1} that fingers were just possible. All of the experiments with R_ρ near 2.5 showed convecting reservoirs but the finger length grew roughly like $t^{1/2}$, just as it did in the experiments in which vertical diffusion completely dominates. Thus, the tendency toward more rapid penetration of the fingers with R_ρ close to 3 tends to be just compensated by the sweeping away of finger ends by convection in the reservoirs. As we noted above, when R_ρ is closer to unity, reservoir convection keeps the fingers short.

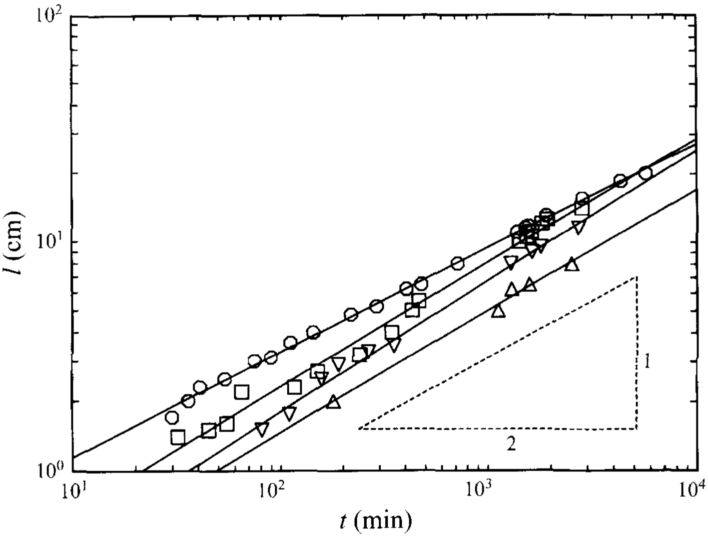

FIGURE 6. Log-log plot of finger length *vs.* time for experiments 3 (circle), 4 (square), 5 (triangle, vertex up), and 6 (triangle, vertex down) all of which started with R_ρ near 2.5. The $t^{1/2}$ power law expected from diffusive growth of the finger zone thickness is shown by the dashed line and the solid lines are least-square power law fits to the data points for each experiment.

3.2. *Salt and sugar fluxes*

Salt and sugar are redistributed vertically by molecular diffusion and advected by the fingers. The diffusive process transports salt more rapidly than sugar, but since the system is driven by the sugar distribution, fingers transport more sugar than salt. Hence, R_f, the density flux ratio of salt to sugar, exceeds unity for pure diffusion and is less than unity when the transport is due to fingers.

In experiments 2 to 10 we estimated the net salt and sugar fluxes into the layers by evaluating $\alpha\delta T/\delta t$ and $\beta\delta S/\delta t$ in each reservoir. Here, $\delta T(\delta S)$ is the measured change of salt (sugar) concentration over the time interval, δt, between measurements. In previous work (e.g. Griffiths & Ruddick 1980) measurements of these T and S fluxes in a salt fingering system were presented as the contribution to the density flux of the more slowly diffusing component (sugar in the present case), βF_S, normalized by $(\beta \Delta S)^{4/3}$ (the 4/3 law) and in terms of the buoyancy flux ratio, $R_f = \alpha F_T/\beta F_S$. Figure 7 shows such a plot of $C = \beta F_S/(\beta\Delta S)^{4/3}$ *vs.* R_ρ for the present experiments. This plot is equivalent to figure 4 of Griffiths & Ruddick (1980) and shows the power-law coefficient to the 4/3 law suggested by these authors. The results from the present experiments follow (but are offset from) the previous results. This offset may be due to the different ranges of $\beta\Delta S$ used in the two sets of experiments. The points from individual experiments tend to approach the average slope but then decrease much more rapidly at higher R_ρ, consistent with the the convective flux vanishing at $R_\rho = \tau^{-1}$. The obvious exception to this clear trend is experiment 10 (crosses on the figure) which started at a low value of R_ρ and had a small $\beta\Delta S$. Note that figure 7 does not test the $\beta\Delta S^{4/3}$ dependence of the sugar flux since any variation in the 4/3 exponent could easily be masked by the experimental scatter. In fact, the very rapid decrease in the fluxes with R_ρ makes it difficult to test the 4/3 power law in this type of system.

Given the different types of finger behaviour already described, it is not surprising

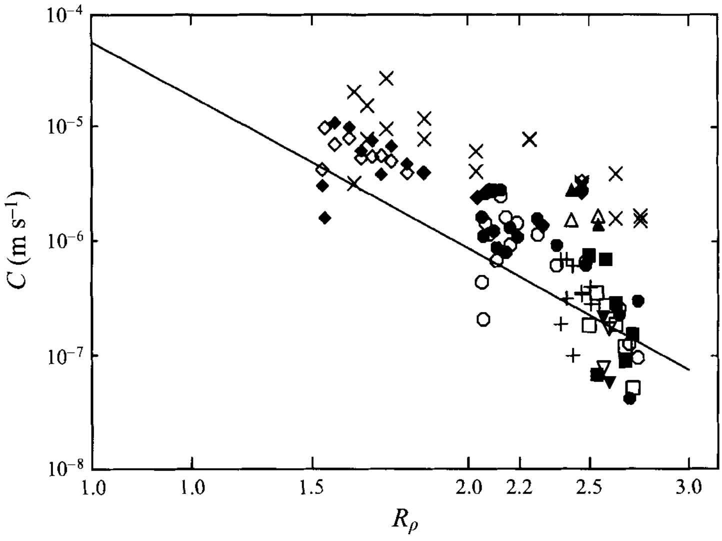

FIGURE 7. Comparison of sugar flux in the present experiments with measurements of Griffiths & Ruddick (1980). The coefficient C is equal to $\beta F_S/(\beta \Delta S)^{4/3}$. The solid line corresponds to $C = 5.5 \times 10^{-5} R_\rho^{-6}$ m s^{-1} which fits the results given by Griffiths & Ruddick (in their experiments R_ρ was in the range $1.03 \leqslant R_\rho \leqslant 2.2$ and the initial $\beta \Delta S$ was in the range $58 \leqslant \rho\beta\Delta S \leqslant 107$ kg m^{-3}). Symbols are: circle, exp. 2; square, exp. 3; triangle (vertex down), exp. 4; triangle (vertex up), exp. 7; diamond, exp. 8; plus, exp. 9; and cross, exp. 10. Where possible, results from the lower layer in this and subsequent figures are shown by filled symbols.

that a single power law does not encompass all the variation in βF_S with R_ρ. Griffiths & Ruddick also found deviations from their suggested power law when R_ρ was close to 1. Therefore, we suggest that the 4/3 power law should not be applied indiscriminately to salt finger convection.

The results for the buoyancy flux ratio R_f from experiments 2, 3, 4, and 8 are presented as the average value from each experiment (figure 8), along with the range of R_ρ covered during the experiment. Like the flux ratios presented by Griffiths & Ruddick the values on figure 8 are not corrected for any contribution from molecular diffusion. The measured values of R_f are lower than the average value of Griffiths & Ruddick, which is shown by the dashed line at $R_f = 0.88$.

In the remainder of this section we focus on how vertical molecular diffusion becomes a significant mechanism of transport through salt fingers at high R_ρ. Clearly this part of the flux is not accounted for in the dimensional reasoning which lead to the 4/3 law. We begin by looking at experiment 11, where R_ρ was so close to τ^{-1} that diffusion dominated advective transport in the fingers. In this case the distribution of salt can be described by the one-dimensional diffusion equation (D is diffusivity)

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial z^2}.$$

In the idealized two-layer initial state the solution for the salt gradient, T_z, is

$$T_z = c(Dt)^{1/2} e^{-z^2/4Dt}$$

where c is constant so the vertical distribution of T_z is Gaussian.

We have fitted a Gaussian profile to each measured vertical profile of T_z for experiment 11 and then used the profiles to plot $\log(1/4Dt)$ *vs.* $\log(t)$. The result is

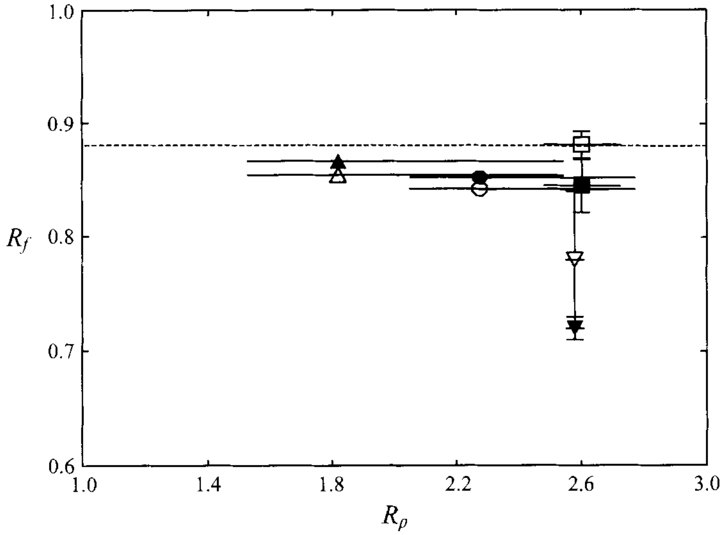

FIGURE 8. Averaged flux ratios based on measurements of reservoir density and conductivity. The horizontal bars show the range of R_ρ used in determining each point. Symbols are: circle, exp. 2; square, exp. 3; triangle (vertex down), exp. 4; triangle (vertex up), exp. 8. Where possible, results from the lower layer are shown by filled symbols. Experiments 7 and 9 were adversely affected by evaporation from the upper layer so no useful average value of R_f could be found. The horizontal dashed line shows $R_f = 0.88$, the value found by Griffiths & Ruddick for $R_\rho = 2.0$.

shown in figure 9. Ignoring the first two points (the initial profile is not strictly two layer so the Gaussian profile is not accurate at low t) a least-squares fit to the data gives an exponent of $-1.007(\pm 0.006)$ and an intercept of $1/4D$. From these results we deduce $D = 1.43(\pm 0.10) \times 10^{-9}\ \mathrm{m^2 s^{-1}}$, in good agreement with the molecular diffusion coefficient for salt. That molecular diffusion was responsible for the thickening of the finger zone was checked by differencing successive salinity profiles to approximate the time rate of change of salinity at any level and numerically integrating these differences in the vertical to compute the salt flux. The resulting maximum salt flux from each pair of profiles was very close to that given by the product of the average maximum gradient from the two profiles and the molecular diffusivity of salt.

Hence, we have an interface profile that grows diffusively. But the system is not purely diffusive because salt–sugar fingers extend through the profile. Thus, the fingers serve to distribute the sugar as if it were being diffused vertically with the diffusivity of salt. However, neither sugar nor salt is deposited beyond the extremities of the salt-diffusion region, and since we use the rate of change in the reservoir concentrations to determine the finger fluxes, the latter vanish for this case.

In the runs with $R_\rho < \tau^{-1}$ we could use the measured profile to estimate the diffusive contribution to the overall salt flux as $\kappa_T T_z$ evaluated at z=0. This is a correct estimate of the *maximum* diffusive flux. However, it will be equivalent to the diffusive flux which is injected into the reservoirs only for those cases in which T_z is constant throughout the interface. In such a case the diffusive contribution could be subtracted from the total flux inferred from the change in the properties of the upper- and lower-layer reservoirs to enable the actual finger flux to be isolated. However, in most of the profiles the mean gradient in the finger region was not independent of z so that some undetermined part of the diffusive flux goes to changing the gradient within the finger zone rather than the properties in the reservoirs. This prevents the

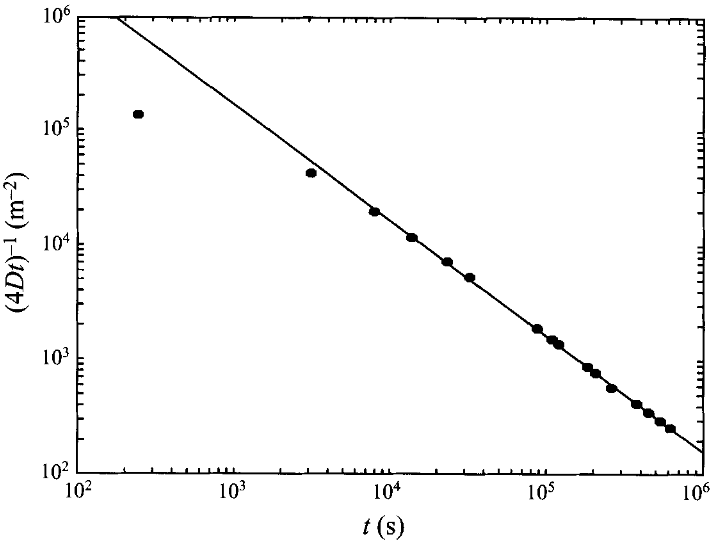

FIGURE 9. log $1/4Dt$ *vs.* log t is essentially linear for experiment 11 which shows that the penetration of the finger length is due to diffusion.

diffusive component of the overall flux from being isolated accurately. An alternative approach of using the average gradient $\delta T/h$ could underestimate the diffusive flux into the reservoirs.

As an example of the possible role of vertical diffusion in experiments at moderate R_ρ, figure 10 shows the salt and sugar fluxes *vs.* time and for experiment 2. During the 5 to 6 day run R_ρ increased from about 2 to 2.75. δT and δS are small initially and could not be evaluated accurately so the data are noisy. The fluxes obtained from the upper and lower reservoirs are significantly different, particularly near the beginning of the experiment. The large error bars are shown for sugar. If the difference between the upper- and lower-layer fluxes is significant, strong asymmetry in the transport through the fingers must have occurred.

Also plotted as solid curves in figure 10 are the diffusive fluxes, $\kappa_T \alpha \partial T/\partial z$ and $\kappa_S \beta \partial S/\partial z$, at z=0, where the T and S gradients were maximum. The salt gradient is determined from the conductivity profile and the sugar gradient is assumed to be $(1/R_\rho)\alpha \partial T/\partial z$.

Except for the first few points, for which the measured δT and δS are not reliable, figure 10 shows that the total flux of salt into the reservoirs is larger than but of the same order of magnitude as the diffusive flux. As time (and R_ρ) increases, however, the maximum diffusive flux becomes as large as the total flux. As described above, the total sugar flux is calculated from measured estimates of $\delta S/\delta t$ in the two reservoirs but the diffusive flux is determined from the estimated salt flux together with the value of R_ρ for each time. This diffusive sugar flux is always substantially smaller than the total even though its value is overestimated. These results show how the component of the fluxes due to vertical diffusion down the mean T and S gradients in the fingers becomes significant at higher R_ρ. This is indeed as would be expected as the fingers approach a regime in which the salt transport within the finger region becomes totally dominated by vertical diffusion.

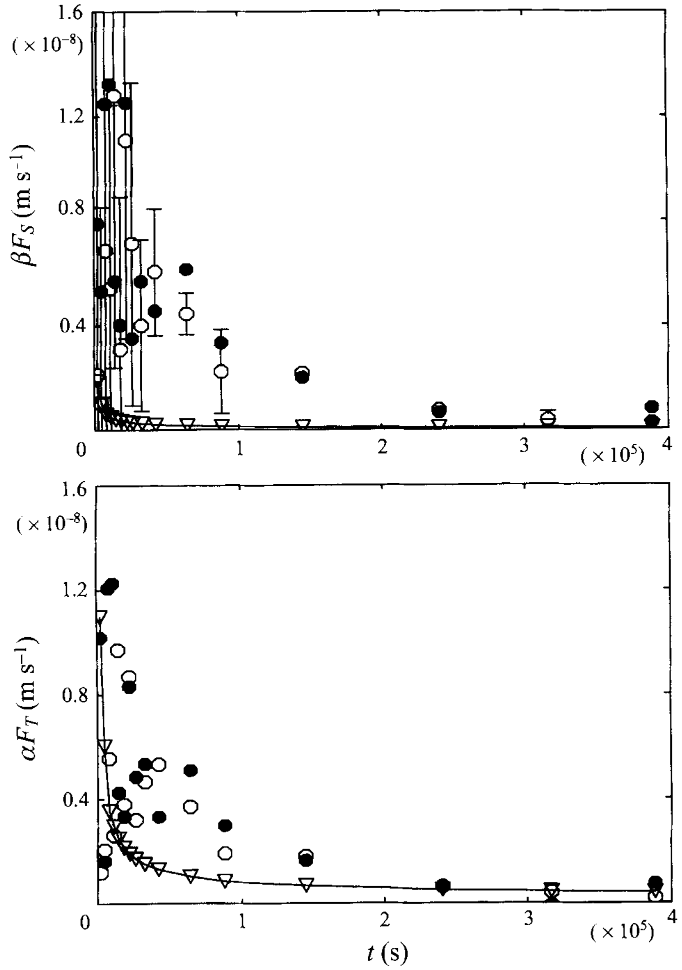

FIGURE 10. Salt and sugar fluxes *vs.* time as determined by the concentration changes in the reservoirs for experiment 2. Both the total (circles) and the diffusive fluxes (triangles) are shown.

4. Conclusions

The qualitative features of these salt–sugar experiments are familiar. The initial evolution of fingers from two layers, followed by increasing finger length and subsequent erosion by convective motions have often been observed. We have documented that the evolution of the fingers can be broken into two apparent regimes. First, in the cases with low R_ρ the fingers initially grow rapidly, overshoot the thickness of the gradient region and then are eroded by convection. They then lengthen smoothly (linearly with time) as the system runs down. Second, when R_ρ is initially closer to τ^{-1}, the fingers evolve more monotonically but the length grows like $t^{1/2}$ when the system is diffusive and slower than $t^{1/2}$ (or not at all) when convection limits the length. The emphasis in these experiments has been on this second regime, the cases with high R_ρ when the fingers become long, as they are generally assumed to be when the process is modelled. The extreme case is for $R_\rho = \tau^{-1}$ when the system is completely dominated by diffusion and reservoir convection is absent.

The vertical sugar fluxes have been determined from the changes with time of the sugar concentrations of the reservoirs. Our values for the density flux ratio are generally smaller than those of earlier experimenters because the longer fingers of the present experiments allow for a more effective short-circuiting of the stabilizing salt stratification by horizontal diffusion. At smaller values of R_ρ the flux ratio is close to the values measured in earlier experiments. The finger width in the experiments with low R_ρ need not be at the value corresponding to maximum growth rate (or to any other optimizing criterion) since the sugar can form fingers when only a small part of the salt anomaly is diffused laterally when the system is not far from unstable stratification. For large R_ρ most of the salt anomaly has to be diffused laterally before the sugar makes the fluid unstable so the width has to be closer to the optimum value.

One conclusion that we draw from the present study is that the configuration of long fingers so often invoked for modelling salt fingers is pertinent mainly for rather diffusive systems. Furthermore, the flux ratio for such systems tends to be smaller than those from earlier experiments because the stabilizing anomaly has time to diffuse laterally so that less of it is convected vertically by the fingers. Since most laboratory experiments have dealt with rather low values of R_ρ (the oceanographically 'realistic' case), the flux ratios that emerge are rather high – certainly higher than is consistent with long, quasi-steady fingers. It is likely, therefore, that the low flux ratios predicted by the theories are closer to being correct than has generally been recognized. If an error has been made, it is probably in comparing results from experiments conducted with initially low R_ρ and high ΔS to theories based on a configuration consistent with high R_ρ and low ΔS.

Concerning the S flux (sugar in our case) we have seen that the fingers evolve in quite different ways depending on the stability ratio of the system. Given this result the pertinence of global descriptions of the finger fluxes, like the 4/3 law, must be doubtful. This is certainly not a new point; Griffiths & Ruddick have pointed out departures from that law for small values of R_ρ and Schmitt (1987) and Lee & Veronis (1993) found that oceanographic data from the thermohaline staircase east of Barbados are inconsistent with the 4/3 law. However, our results showing different modes of finger evolution now add a high-R_ρ regime which breaks the requirement of similarity of the salt finger zone. The 4/3 law was originally suggested by Turner (1967) as a plausible extension of results from turbulent thermal convection so the fact that it is not applicable to the very different physics of double-diffusive convection should come as no surprise.

The obvious extension of the present results is to consider the role of vertical diffusion in the evolution of heat salt rather than salt sugar fingers, and to find the stability ratio at which vertical diffusion dominates advection in the fingers. The much smaller value of τ for heat and salt compared to that for salt and sugar suggests that the transition to the diffusively dominated regime of salt fingering may occur much earlier in the range of R_ρ available to heat–salt fingers. Perhaps this may be in part why fingers appear to be important in the ocean only at low R_ρ. At higher stability ratios, while salt fingers may be present in the ocean, they are likely to be passive in terms of heat transport, while perhaps still supporting a significant salt flux.

We wish to thank Stewart Turner and the Research School of Earth Sciences at the Australian National University for allowing us to use the facilities in the Geophysical Fluid Dynamics Laboratory. Tony Beasley built the tank that was used. Special thanks go to Derek Corrigan for his help with all aspects of the experiment. G.V. is grateful to Stewart Turner and Ross Griffiths for the opportunity to spend four months at the GFD laboratory. Barry Ruddick made some very penetrating comments that helped us to improve an earlier draft of this article. The research was partly supported by Grants NSF OCE-9010662 and 9312523 of the US National Science Foundation.

REFERENCES

GRIFFITHS, R. W. & RUDDICK, B. R. 1980 Accurate fluxes across a salt-sugar finger interface deduced from direct density measurements. *J. Fluid Mech.* **99**, 85–95.

HEAD, M. J. 1983 The use of miniature four-electrode conductivity probes for high resolution measurements of turbulent density or temperature variations in salt-stratified water flows. PhD dissertation, University of California, San Diego, 211pp.

HOLYER, J. Y. 1984 The stability of long, steady, two-dimensional salt fingers. *J. Fluid Mech.* **147**, 169–185.

HOWARD, L. N. & VERONIS, G. 1987 The salt-finger zone. *J. Fluid Mech.* **183**, 1–24.

HOWARD, L. N. & VERONIS, G. 1992 Stability of salt fingers with negligible diffusivity. *J. Fluid Mech.* **238**, 511–522.

LAMBERT, R. B. & DEMENKOW, J. W. 1972 On vertical transport due to fingers in double diffusive convection. *J. Fluid Mech.* **54**, 627–640.

LEE, J. H. & VERONIS, G. 1993 Inversions of data from the thermohaline staircase in the western tropical North Atlantic. *Deep-Sea Res.* **40**, 1839–1862.

LINDEN, P. 1973 On the structure of salt fingers. *Deep-Sea Res.* **20**, 325–340.

MCDOUGALL, T. J. 1981 Double-diffusive convection with a nonlinear equation of state. *Prog. Oceanogr.* **10**, 91–121.

MCDOUGALL, T. J. & TAYLOR, J. R. 1984 Flux measurements across a finger interface at low values of the stability ratio. *J. Mar. Res.* **42**, 1–14.

RUDDICK, B. R. & SHIRTCLIFFE, T. 1979 Data for double diffusers: Physical properties of aqueous salt-sugar solutions. *Deep-Sea Res.* **26**, 775–787.

SCHMITT, R. W. 1979 Flux measurements on salt fingers at an interface. *J. Mar. Res.* **37**, 419–436.

SCHMITT, R. 1987 The Caribbean Sheets and Layers Transects (C-SALT) Program. *Eos, Trans Am. Geophys. Union* **68**(5), 57–60.

STERN, M. E. 1960 The 'salt fountain' and thermohaline convection. *Tellus* **12**, 172–1175.

STERN, M. E. 1969 The collective instability of salt fingers. *J. Fluid Mech.* **35**, 209–218.

STERN, M. E. 1975 *Ocean Circulation Physics.* Academic Press.

STERN, M. E. 1976 Maximum buoyancy flux across a salt finger interface. *J. Mar. Res.* **34**, 95–110.

STERN, M. E. & TURNER, J. S. 1969 Salt fingers and convecting layers. *Deep-Sea Res.* **16**, 497–511.

TAYLOR, J. R. & P. BUCENS, P. 1989 Laboratory experiments on the structure of salt fingers. *Deep-Sea Res.* **36**, 1675–1704.

TURNER, J. S. 1967 Salt fingers across a density interface. *Deep-Sea Res.* **14**, 499–611.

VERONIS, G. 1987 The role of the buoyancy layer in determining the structure of salt fingers. *J. Fluid Mech.* **180**, 327–342.